Analysis of Planar Circuit Discontinuities using the Quasi-Static Space-Spectral Domain Approach

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Abstract

A new deterministic quasi-static method is presented for the analysis of arbitrarily shaped planar circuit discontinuities including air bridges. Using the basic principles of the space-spectral domain approach (SSDA), this new method leads to an algebraic matrix equation for the discontinuity s-parameters. The resulting algorithm is computationally very efficient. Results are presented for microstrip and CPW discontinuities as well as CPW air bridges.

Summary

Accurate and computationally efficient characterization of planar circuit discontinuities is important in the design of monolithic microwave MIC's (MMIC's) and Miniature Hybrid MIC's (MHMIC's). Therefore, an increasing number of computer-aided design tools available today is based on rigorous full-wave analysis methods. Although these methods provide the highest degree of accuracy, they often require complicated analytical formulations, large amounts of computer memory and long computer run-time. In comparison, quasi-static numerical techniques are traditionally faster than full wave techniques, in particular on serial computers widely used today. For some applications their accuracy can even rival that of full-wave techniques and therefore they are still very useful engineering design tools.

In this paper we introduce a new deterministic quasi-static method which is not only computationally very efficient, but also very flexible in that a wide range of planar circuit discontinuities, including CPW air bridges, can be treated. The method is based on the space-spectral domain approach (SSDA) which combines the advantages of the one-dimensional spectral domain approach (SDA) [3] in x-direction with that of the one-dimensional method of lines (MoL) [4] in z-direction (the propagation direction). This method was first introduced by Wu and Vahldieck [2] and has since been improved by the authors to calculate the discontinuity s-parameters directly from an eigenvalue equation. The full-wave SSDA is a very general algorithm taking into account the dispersive effect in a transmission line. For some applications, however, in particular for low-dispersive transmission line structures in which discontinuities can be described by equivalent capacitances, the full-wave SSDA is a kind of computational "overkill". For these applications we have developed a new deterministic quasi-static version of the SSDA.

The new method utilizes the basic idea of the SSDA, but avoids solving an eigenvalue problem. To minimize errors in the calculation of the capacitance parameters, the excess charge density [5] has been used and calculated in the space-spectral domain in one step via Galerkin's method. This approach leads to an algebraic equation (deterministic) for the s-parameters of the discontinuities and is computationally very stable, requires little memory space and is very fast on serial computers.
Although the method is capable of treating arbitrarily shaped planar circuit discontinuities as shown in Fig.2, we are concentrating in the analysis first of all on commonly used discontinuities such as shown in Fig.1. Since the equivalent circuits of these discontinuities exist

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mainly of capacitances (Fig.1), their values can be derived from the excess charge density on the conductors. Knowing the capacitance values leads directly to the s-parameters of the discontinuity. A comparison with full-wave techniques and measurements shows that, as long as the transmission line is low-dispersive, the resulting s-parameters can be accurate up to 40 GHz.

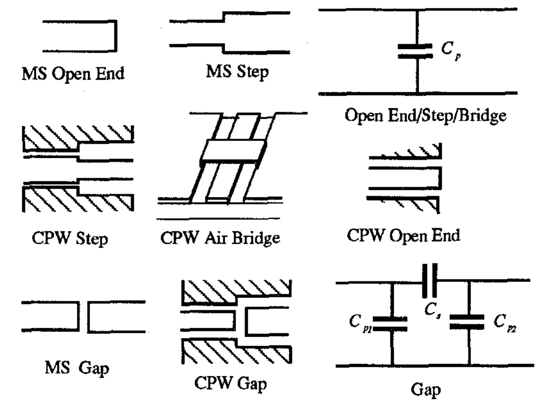

Fig.1 Planar circuit discontinuities and equivalent circuit

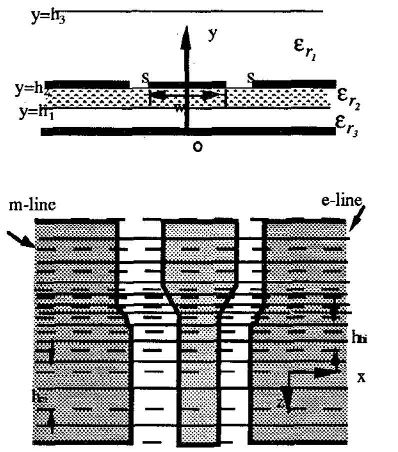

Fig.2 Discretization of a discontinuity

Theory

Quasi-static modeling is based on the 3D Laplace's equation $\nabla^2\Phi = 0$ which contains the three spatial variables x,y,z. The three-layer structure in Fig.2 is *sliced* in the x-y-plane at each z-coordinate. The goal is to reduce this three-dimensional differential equation to a decoupled one-dimensional form, which depends only on the spatial variable y. The solutions are then simple transmission line equations. The procedure is as follows:

By transforming the electric potential Φ into $\tilde{\Phi}$ via a Fourier transform along the x-direction (Fig.2), the x-variable in the Laplace equation is replaced by the Fourier variable α. Further, discretizing the z-direction leads to the vector $\overline{\Phi}$ (in the spectral domain) for all lines in z-direction and, hence, also the z-variable is eliminated. Following the procedure of the MoL, we transform $\overline{\Phi}$ into $\overline{\Psi}$ by using an orthogonal transformation to decouple Laplace's equation. The Laplace equation is now reduced to a one-dimensional differential equation in the spectral domain with only one remaining variable (y).

$$\frac{\partial^2\overline{\Psi}}{\partial y^2}-\gamma^2\overline{\Psi}=0 \quad , \quad \gamma^2=\alpha^2+\delta^2 \tag{1}$$

where

$$[T]^t\left[D_{zz}\right][T]=\delta^2$$

δ is a diagonal matrix and α the spectral variable. Solutions to the above equation can be expressed in terms of the sum of hyperbolic functions. This makes it possible to transform the boundary conditions from the top and bottom metallic cover onto the strip and groundplane and to formulate a characteristic equation in the spectral domain, in which the transformed electrostatic potential is related to the transformed charge density for each line. Applying the reverse orthogonal transform into the original domain, couples all lines in z-direction and leads to the following characteristic equation in the spectral domain:

$$\overline{\Phi}=\overline{G}(\alpha)\overline{\sigma} \tag{2}$$

where

$$\overline{G}(\alpha)=\left[r^h\right]^{-1}[T]^t\left(g(\gamma)-g(\alpha)\right)^{-1}[T]\left[r^h\right]$$

$$g(\beta)=diag\left\{\frac{\varepsilon_{r_2}\beta}{tanh\left(\beta h_2\right)}+\frac{\varepsilon_{r_1}\beta}{tanh\left(\beta h_1\right)}-\frac{\left(\frac{\varepsilon_{r_2}\beta}{sinh\left(\beta h_2\right)}\right)^2}{\frac{\varepsilon_{r_2}\beta}{tanh\left(\beta h_2\right)}+\frac{\varepsilon_{r_3}\beta}{tanh\left(\beta h_3\right)}}\right\}$$

The charge density is expanded as the sum of basis functions σ_i. Sinusoidal functions [3] are used here. With Parseval's theorem

$$\int\Phi(\alpha)\sigma_i(\alpha)d\alpha=2\pi V\sigma_i(\alpha=0) \tag{3}$$

and after applying Galerkin's technique in the spectral domain for each discrete line, the coefficients of the excess charge density basis functions can be obtained by one matrix inversion. The capacitance Cp1,Cp2,Cs are then calculated from C=Q/V assuming different excitation voltages V (even mode V1=1, V2=1, odd mode V1=1, V2=-1) at both ports of the strip. V=0 is chosen for the ground conductor. The s-parameters can be derived by using network theory.

Results and Discussion

The microstrip (MS) and CPW open end, gap and step in width as well as the CPW air bridge are analyzed with this method. Results are shown in Fig.3. The calculated capacitances or s-parameters compare well with measurement and full-wave analysis results from the literature [1, 5 - 12]. The only discrepancy we could find was with the full-wave analysis results of [11], in particular the phase calculations published in that paper. The computation time of the quasi-static SSDA is typically 3 seconds per frequency on a SUN SPARC-2 station.

Conclusion

A new deterministic quasi-static space-spectral domain approach (SSDA) has been developed to analyze planar circuit discontinuities.This new approach extends the SSDA to calculate quasi-static capacitances and s-parameters of arbitrarily shaped planar discontinuities. The discontinuity parameters are derived from an algebraic matrix equation instead from an eigenvalue matrix. A comparison of the results with other methods and measurements shows excellent agreement up to 40GHz and more. The advantages of this new method makes it an attractive CAD tool for engineering applications.

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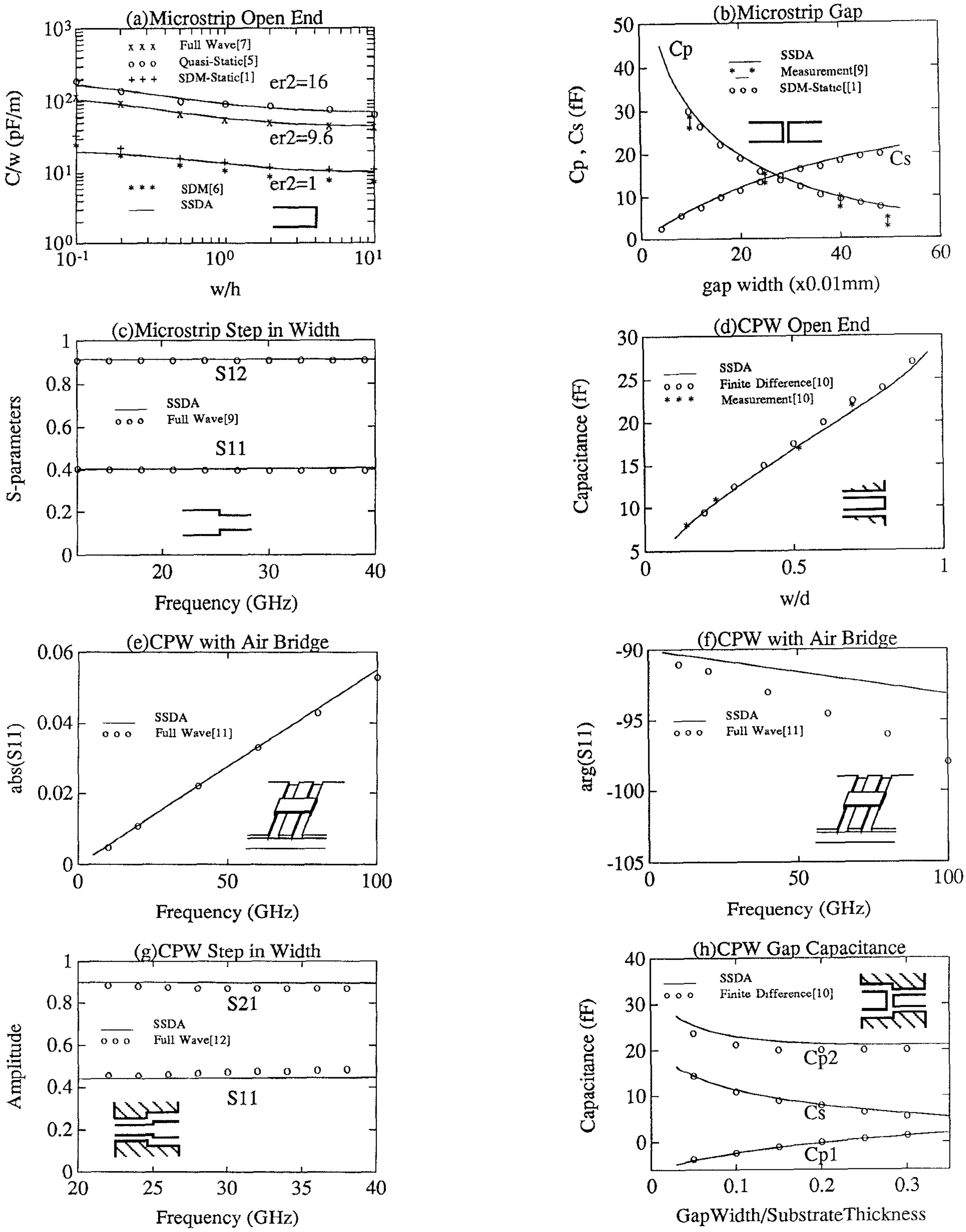

Fig.3 Results of Numerical Analysis